

## IM

## INTERNATIONAL MATHEMATICS OLYMPIAD



Shraddha Singh



#  

# F-2/16, Ansari road, Daryaganj, New Delhi-110002 <br> 匹 23240026, 23240027• Fax: 011-23240028 <br> Email: info@vspublishers.com •Website: www.vspublishers.com 

## Regional Office : Hyderabad

5-1-707/1, Brij Bhawan (Beside Central Bank of India Lane)
Bank Street, Koti, Hyderabad - 500095
उ 040-24737290
E-mail: vspublishershyd@gmail.com

## Branch Office : Mumbai

Jaywant Industrial Estate, 1st Floor-108, Tardeo Road
Opposite Sobo Central Mall, Mumbai - 400034
© 022-23510736
E-mail: vspublishersmum@gmail.com

## Follow us on: 3 in

## © Copyright: $\mathcal{V}_{\&} S_{\text {Punduiln }}$ <br> ISBN 978-93-505795-6-5

## DISCLAIMER

While every attempt has been made to provide accurate and timely information in this book, neither the author nor the publisher assumes any responsibility for errors, unintended omissions or commissions detected therein. The author and publisher makes no representation or warranty with respect to the comprehensiveness or completeness of the contents provided.
All matters included have been simplified under professional guidance for general information only, without any warranty for applicability on an individual. Any mention of an organization or a website in the book, by way of citation or as a source of additional information, doesn't imply the endorsement of the content either by the author or the publisher. It is possible that websites cited may have changed or removed between the time of editing and publishing the book.
Results from using the expert opinion in this book will be totally dependent on individual circumstances and factors beyond the control of the author and the publisher.
It makes sense to elicit advice from well informed sources before implementing the ideas given in the book. The reader assumes full responsibility for the consequences arising out from reading this book.
For proper guidance, it is advisable to read the book under the watchful eyes of parents/guardian. The buyer of this book assumes all responsibility for the use of given materials and information.
The copyright of the entire content of this book rests with the author/publisher. Any infringement/transmission of the cover design, text or illustrations, in any form, by any means, by any entity will invite legal action and be responsible for consequences thereon.

## Publisher's Note

V\&S Publishers, after the grand success of a number of Academic and General books, is pleased to bring out a series of Mathematics Olympiad books under The Gen X series generating Xcellence in generation $X$ - which has been designed to focus the problems faced by students. In all books the concepts have been explained clearly through examples, illustrations and diagrams wherever required. The contents have been developed to meet specific needs of students who aspire to get distinctions in the field of mathematics and want to become Olympiad champs at national and international levels.
To go through Maths Olympiad, the students need to do thorough study of topics covered in the Olympiad syllabus and those covered in the school syllabus as well. The Olympiads not only tests subjective knowledge but Reasoning skills of students also. So students are required to comprehend the depth of concepts and problems and gain experience through practice. The Olympiad check efficiency of candidates in problem solving. These exams are conducted in different stages at regional, national, and international levels. At each stage of the test, the candidate should be fully prepared to go through the exam. Therefore, this test requires careful attention towards comprehension of concepts, thorough practice, and application of rules.
While other books in market focus selectively on questions or theory; V\&S Maths Olympiad books are rather comprehensive. Each book of this series has been divided into four sections namely Mathematics, Logical Reasoning, Achievers section, Model Papers. The theory has been explained through solved examples. To enhance the problem solving skills of candidates, Multiple Choice Questions (MCQs) with detailed solutions are given at the end of each chapter. Two Mock Test Papers have been included to understand the pattern of exam. A CD containing Study Chart for systematic preparation, Tips \& Tricks to crack Maths Olympiad, Pattern of exam, and links of Previous Years Papers is accompanied with this book. The books are also useful for various other competitive exams such as NTSE, NSTSE, and SLSTSE as well.
We wish you all success in the examination and a very bright future in the field of mathematics.
All the best

## Contents

## SECTION 1 : MATHEMATICAL REASONING

1. Number System ..... 9
2. Roman Numbers ..... 27
3. Operations on Numbers ..... 36
4. Decimals and Fractions ..... 50
5. LCM and HCF ..... 69
6. Ratio and Proportion ..... 86
7. Measurement ..... 100
8. Temperature ..... 111
9. Money ..... 118
10. Area, Perimeter and Volume ..... 124
11. Geometrical Shapes and Angles ..... 131
SECTION 2: LOGICAL REASONING
12. Series and Pattern ..... 159
13. Analogy ..... 171
14. Odd One Out ..... 180
15. Coding and Decoding ..... 184
16. Number Ranking and Alphabet Test ..... 201
17. Direction Sense Test ..... 212
18. Mirror and Water Images ..... 222
19. Pictorial Representation of Data ..... 233SECTION 3 : ACHIEVERS SECTION
今 High Order Thinking Skills ..... 255
SECTION 4 : MODEL PAPERS
$\diamond$ Model Test Paper - 1 ..... 259
↔ Model Test Paper - 2 ..... 266

# Section 1 Mathematics Reasoning 

## Unit-1 : Number System



Learning Objectives: In this unit, we will learn about:

- Number System
- Number Sense
- Types of Numbers
- Fractions
- Integers
- Place Value
- Rules of Divisibility


## Number System

The number system that we use in our everyday life is called decimal system. This is because there are 10 digits ( $0,1,2,3,4,5,6,7,8$, and 9 ). Numbers are written using these digits.

## Number Sense

Number sense refers to a student's fluidity and elasticity with numbers. He/She has sense of what number mean, understands symbolic representation, is able to perform mental math, understands their relationships to one another and can use those numbers in real world problems.

## Formation of numbers from given digits

The greatest and smallest numbers, without repetition, can be formed using any number of digits by arranging them in the descending and ascending order respectively.
For example, if we form a 4-digit number from the digits, 9, 2, 7 and 3, then
Greatest number $=9732$
Smallest number $=2379$
When repetition of digits is allowed, then the greatest number can be formed by writing the greatest digit as many times as the number is required. Similarly, the smallest number can be formed by writing the smallest digit as many times as required.

Greatest number (with repetition) $=9999$
Smallest number (with repetition) $=2222$

## Types of Numbers

Now let's classification of Numbers is done.


## Real Number Line

Set of all numbers that can be represented on the number line are called real numbers. e.g. $4,-8,0, \frac{9}{11}$ etc.

$$
\left.-\infty \frac{\mid}{\mid} \left\lvert\, \begin{array}{llllllllll}
\mid & \mid & \mid & \mid & \mid & \mid & \mid & \mid & \mid & \mid \\
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3
\end{array}\right.\right)
$$

## Rational Numbers

A number that can be represented in the form $\mathrm{p} / \mathrm{q}$, where p and q are integers and q is not zero. Example : 2/3, 1/10, $8 / 3$ etc. They can be finite decimal numbers, whole numbers, integers, fractions.

## Irrational Numbers

A number that cannot be represented in the form $\mathrm{p} / \mathrm{q}$, where p and q are integers and q $\neq$ zero. An infinite non recurring decimal is an irrational number. Example : $\sqrt{2}, \sqrt{5}, \sqrt{7}$ and $\pi(\mathrm{pie})=3.1416$.
The rational numbers are classified into integers and fractions

## Integers

The set of numbers on the number line, with the natural numbers, zero and the negative numbers are called integers, $I=\{\ldots . .,-3,-2,-1,0,1,2,3, \ldots \ldots$.


## Fractions

A fraction denotes part or parts of an integer. For example 1/6, which can represent 1/6th part of the whole.

## Types of Fraction

1. Common Fractions : The fractions where the denominator is not 10 or a multiple of it. Example : 2/3, 4/5 etc.
2. Decimal Fractions: Decimals are a type of fractional number. The decimal 0.5 represents the fraction 5/10. The decimal 0.25 represents the fraction 25/100. Decimal fractions always have a denominator based on a power of 10. The fractions where the denominator is 10 or a multiple of 10 .
Example : 7/10, 9/100, etc.


Any rational number (that is, a fraction in lowest terms) can be written as either a terminating decimal or a repeating decimal. Just divide the numerator by the denominator. If you end up with a remainder of 0 , then you have a terminating decimal. Otherwise, the remainders will begin to repeat after some point, and you have a repeating decimal.
3. Proper Fractions : The fractions where the numerator is less than the denominator. Example : $3 / 4,2 / 5$ etc. its value is always less than 1
4. Improper Fractions: The fractions where the numerator is greater than or equal to the denominator.
Example : 4/3, 5/3 etc. Its value is always greater than or equal to 1 .
5. Compound fraction : A fraction of a fraction is called a compound fraction Example : $3 / 5$ of $7 / 9=3 / 5 \times 7 / 9=21 / 45$
6. Complex Fractions : The combination of fractions is called a complex fraction. Example : (3/5)/ (2/9)
7. Mixed Fractions : A fraction which consists of two parts, an integer and a fraction. Example: 3 ½, 6 3/4
Q.1. Express $27 / 8$ as a mixed fraction

Ans. Divide the numerator by denominator; note the multiplier, whatever remainder is left divide it with the original denominator. For $27 / 8,24 / 8=3$, and remainder left is 3 , therefore $33 / 8$ is the mixed fraction
Q.2. Express $357 / 17$ as an improper fraction.

Ans. Here we need to multiply the denominator with the non-fraction part and add it to numerator and using same denominator.
For $357 / 17==602 / 17$

## Integers

The integers are classified into negative numbers and whole numbers
Negative Numbers : All the negative numbers are on the number line, $\{\ldots . .-3,-2,-1\}$
Natural Numbers: The counting numbers 1, 2, 3, 4, 5...... are known as natural numbers, $N=\{1,2,3,4,5 \ldots .$.$\} . The natural numbers along with zero make the set of the whole$ numbers.
Whole Numbers : The set of all positive numbers and 0 are called whole numbers,

$$
W=\{0,1,2,3,4 \ldots \ldots . .\}
$$

Even Numbers : The numbers divisible by 2 are even numbers. e.g., 2, 4, 6,8,10 etc. Even numbers can be expressed in the form 2 n , where n is an integer other than 0 .
Odd Numbers : The numbers not divisible by 2 are odd numbers. e.g. 1, 3, 5, 7, 9 etc. Odd numbers are expressible in the form $(2 n+1)$, where $n$ is an integer other than 0 .

Prime Numbers : The numbers that have only two factors unity and number itself are called prime factors. e.g. $\{2,3,5,11,13,17,19\}$.
Composite Numbers : A composite number has other factors besides itself and unity .e.g. $8,72,39$ etc. A real natural number that is not a prime number is a composite number.

## Did you know?

- The only even prime number is 2
- 1 is neither a prime nor a composite number.
- If p is a prime number then for any whole number $\mathrm{a}, \mathrm{ap}-\mathrm{a}$ is divisible by p .
- $2,3,5,7,11,13,17,19,23,29$ are first ten prime numbers (should be remembered)
- Two numbers are supposed to be co-prime if their HCF is 1 , e.g. $3 \& 5,14 \& 29$ etc. are coprime numbers.
- A number is divisible by ab only when that number is divisible by each one of a and $b$, where $a$ and $b$ are co prime.
- To find a prime number, check the rough square root of the given number and divide the number by all the prime numbers lower than the estimated square root.
- All prime numbers can be expressed in the form $6 n-1$ or $6 n+1$, but all numbers that can be expressed in this form are not prime.
Prime Factors : The composite numbers express in factors, wherein all the factors are prime. To get prime factors we divide number by prime numbers till the remainder is a prime number. All composite numbers can be expressed as prime factors, for example prime factors of 150 are $2,3,5,5$.
A composite number can be uniquely expressed as a product of prime factors.

$$
\text { e.g. } \begin{aligned}
12 & =2 \times 6=2 \times 2 \times 3=2^{2} \times 3^{1} \\
20 & =4 \times 5=2 \times 2 \times 5=2^{2} \times 5^{1} \text { etc. }
\end{aligned}
$$

Perfect Number : If the sum of the divisor of $N$ excluding $N$ itself is equal to $N$, then $N$ is called a perfect number. e.g. $6,28,496$.

## Imaginary Numbers

Numbers that when squared give a negative result. If you square a real number you always get a positive, or zero, result. For example $2 \times 2=4$, and $(-2) \times(-2)=4$ also, so "imaginary" numbers can seem impossible, but they are still useful.
An imaginary number is denoted by bi, where

- $b$ is a real number.
- $i$ is the imaginary unit $\mathrm{i}=\sqrt{-1}$

Example : Is zero a rational number?
Answer: Zero can be written in the form $\mathrm{p} / \mathrm{q}\left(\frac{0}{1}\right)$, where p and q are integers and $\mathrm{q}(=1)$ is not equal to 0 . Therefore, zero is a rational number.

Example : Find six rational number between 3 and 4.
Sol.
Step $1: \frac{3+4}{2}=\frac{7}{2}$
Step 2: $\frac{\frac{7}{2}+4}{2}=\frac{\frac{7+8}{2}}{2}=\frac{15}{2 \times 2}=\frac{15}{4}$
Step 3: $\frac{3+\frac{7}{2}}{2}=\frac{\frac{6+7}{2}}{2}=\frac{\frac{13}{2}}{2}=\frac{13}{2 \times 2}=\frac{13}{4}$
Step $4: \frac{\frac{15}{4}+4}{2}=\frac{\frac{15+16}{4}}{2}=\frac{31}{4 \times 2}=\frac{31}{8}$
Step $5: \frac{3+\frac{15}{4}}{2}=\frac{\frac{12+15}{4}}{2}=\frac{\frac{27}{4}}{2}=\frac{27}{4 \times 2}=\frac{27}{8}$
Step $6: \frac{\frac{13}{4}+4}{2}=\frac{\frac{13+16}{4}}{2}=\frac{\frac{29}{4}}{2}=\frac{29}{4 \times 2}=\frac{29}{8}$
Hence, six rational numbers between 3 and 4 are

$$
\frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}, \frac{31}{8}
$$

You can notice that by calculating averages between two numbers we get a number which is exactly between these two numbers. This way you can go on calculating infinite numbers of numbers.
Example : Find five rational numbers between $3 / 5$ and $4 / 5$.
Sol. Step $1: \frac{\frac{3}{5}+\frac{4}{5}}{2}=\frac{7}{10}$
Step 2: $\frac{\frac{3}{5}+\frac{7}{10}}{2}=\frac{13}{20}$

$$
\underset{90}{\rightleftarrows} \underset{\square g}{\leftrightarrows+5}
$$

Step 3 :

Step $4: \frac{\frac{3}{5}+\frac{13}{20}}{2}=\frac{5}{8}$
Step $5: \frac{\frac{3}{4}+\frac{4}{5}}{2}=\frac{31}{40}$

Comparison of Numbers : If a number has more digits than the other, then it is greater of the two. For example 8542 is greater than 984.

- If two numbers have the same number of digits, then the number with bigger digit on the extreme left is greater. For example 5732 is greater than 2884.
- If two numbers are the same number of digits and the extreme left digits are also the same, then compare the next digits to the right and so on. For example 8342 is greater than 8217 .
Note : Count the digits first, then check th, then ' $\mathrm{H}^{\prime}$ then ' T ' then O , Symbols used for 'is greater than' and 'is less than' are > and < respectively.


## Numerals

Numbers can be written by using different symbols. The numbers represented by particular symbols are known as the digits of the system. The numeral formed by the digits $0,1,2$, $3,4,5,6,7,8$ and 9 are known as Hindu-Arabic numbers. These are called international numbers.

Number system is a way of counting and naming number. Number is an idea where the symbols used to represent the numbers are called numerals.

## Number Names (7 and 8 digit Numbers)

$$
\begin{aligned}
& 10 \text { ones }=1 \text { ten } \\
& 100 \text { ones }=1 \text { Hundred } \\
& 100 \text { is a three digit numeral. } \\
& 1000 \text { ones }=1 \text { thousand } \\
& 10000 \text { ones }=\text { ten thousand } \\
& 100000 \text { ones }=1 \text { Lakh } \\
& 1000000 \text { ones }=10 \text { Lakh } \\
& 1,00,00,000 \text { ones }=1 \text { Crore }
\end{aligned}
$$

$1,00,00,000$ is the smallest number of eight digits. We know 9999999 is the greatest of seven digits. 1,00,00,000 is the successor of 9999999 .

## Place value

Place value of a digit depends on its position in the number. As the digit moves to the left, its value increases. The place value of each digit in 9999 is as follows.


$$
\frac{\text { Expanded form of } 9999}{9999}=9000+900+90+9
$$

$=9$ thousands +9 hundreds +9 tens +9 ones

## Rules of Divisibility

A number is divisible by 2 if the unit's digit is 0 or divisible by 2 . e.g. $8,16,32$ etc.
Divisibility by 3 : A number is divisible by 3 if the sum of digits in the number is divisible by 3. e.g. 49185
$4+9+1+8+5=27$ since 27 is divisible 3 .
$\therefore 49185$ is divisible by 3 .
Divisibility by 4 : A number is divisible by 4 if the numbers formed by the last two digits is divisible by 4 .
e.g. 6032 is divisible by 4 .

Divisibility by 5 : A number is divisible by 5 if the unit's digit in the number is 0 or 5. e.g. 525 and 110 are divisible by 5 .
Divisibility by $\mathbf{9}$ : A number is divisible by 9 is the sum of its digit is divisible by 9 .
Divisibility by 10 : A number is divisible by 10 if the unit's digit is 0 .
Divisibility by 11 : A given number is divisibility by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.


Prime/Composite Number

Example : State if following statements are true or false:
(a) Every natural number is a whole number.
(b) Every integer is a whole number.
(c) Every rational number is a whole number.

Sol.
(a) As natural number is all numbers starting from 1 and the whole number includes zero as well so this statement is true. On the other hand every whole number is not natural number as zero is not a natural number.
(b) Only positive integers are whole numbers. Hence this statement is false.
(c) Rational numbers are not whole numbers as they are not complete. Hence this statement is false.

Example : Write the following in decimal form and comment on their kind of decimal expression.
(a) $\frac{36}{100}=0.36$, as we get zero as remainder at last so it is a terminating decimal.
(b) $\frac{1}{11}=0.09090909$, we don't get zero as remainder and the quotient keep on repeating, so this is non-terminating recurring decimal.
(c) $4 \frac{1}{8}=4.125$, terminating decimal.

Example : Express the following in the form $\mathrm{p} / \mathrm{q}$, where p and q are integers and q is not 0 .
(a) $0 . \overline{6}=\frac{6}{9}=\frac{2}{3}$
(b) $0 . \overline{47}=\frac{47}{99}$
(c) $0.00 \overline{\overline{1}}=\frac{1}{900}$

Put 9 for every non-zero digit in the denominator and zero for zero in the denominator. A fraction in lowest terms with a prime denominator other than 2 or 5 (i.e. co-prime to 10) always produces a repeating decimal. The period of the repeating decimal, $1 p$, where p is prime, is either $\mathrm{p}-1$ (the first group) or a divisor of $\mathrm{p}-1$ (the second group). Examples of fractions of the first group are :

- $1 / 7=0.142857 ; 6$ repeating digits
- $1 / 17=0.0588235294117647 ; 16$ repeating digits
- $1 / 19=0.052631578947368421 ; 18$ repeating digits
- $1 / 23=0.0434782608695652173913 ; 22$ repeating digits
- $1 / 29=0.0344827586206896551724137931 ; 28$ repeating digits
- $197=0.010309278350515463917525773195876288659793814432$ 989690721649484536082474226804123711340206185567 ; 96 repeating digits
Example : What property a rational number must satisfy to have terminating decimal expression.
Sol. If the denominator is either 2 or 5 as its factor then the result will be terminating decimal. As 10 is the product of 2 and 5 so to have terminating decimal 2 or 5 are required. If there is a prime number other than 2 or 5 in the denominator then the decimal can or cannot be terminating.
Example : Which number is represented on the given abacus?
Sol.


The spike corresponding to ones place has 8 beads.
$\therefore$ Digit at ones place $=8$
The spike corresponding to tens place has 0 beads.
$\therefore$ Digit at tens place $=0$
The spike corresponding to hundreds place has 9 beads.
$\therefore$ Digit at hundreds place $=9$
The spike corresponding to thousands place has 4 beads.
$\therefore$ Digit at thousands place $=4$
The spike corresponding to ten thousands place has 7 beads.
$\therefore$ Digit at ten thousands place $=7$
The spike corresponding to lakhs place has 8 beads.
$\therefore$ Digit at lakhs place $=8$
Thus, the number represented on the given abacus is $8,74,908$.

Example : Write the difference between the roman numerals MDCXXIV and MCCCXL in Hindu Arabic numeral?

## Sol.

We know that,
I stands for $1, \mathrm{~V}$ stands for $5, \mathrm{X}$ stands for $10, \mathrm{~L}$ stands for $50, \mathrm{C}$ stands for 100, D stands for 500 and $M$ stands for 1000 .
$\therefore$ MDCXXIV $=1000+500+100+10+10+5-1$

$$
=1625-1
$$

$$
=1624
$$

and MCCCXL $=1000+100+100+100+50-10$

$$
=1350-10
$$

$$
=1340
$$

Required difference $=$ MDCXXIV - MCCCXL

$$
\begin{aligned}
& =1624-1340 \\
& =284
\end{aligned}
$$

Thus, the difference between the roman numerals MDCXXIV and MCCCXL, in Hindu-Arabic numeral, is 284.

Example : Which number should be filled in the box to complete the given number line?


Sol.
Distance between points 150 and $165=(165-150)$ units $=15$ units
It is known that the distance between any two consecutive points on a number line is equal.
$\therefore$ Distance between any two consecutive points on the given number line $=15$ units.
To obtain the first missing value, add 15 to 180 .
$\therefore$ First missing value $=180+15=195$
To obtain the second missing value, add 15 to 210 .
$\therefore$ Second missing value $=210+15=225$
Example : What is the difference between the largest and the smallest 5-digit numbers formed by the digits $0,9,3,7,2$ using each digit only once?
Sol.
The smallest number formed by the digits $0,9,3,7,2$, using each digit only once, is 02379 .
But, 0 cannot be the first digit of a number. So, the positions of 0 and 2 are interchanged.

So, the smallest number formed by the given digits is 20379.
The largest number formed by the digits $0,9,3,7,2$, using each digit only once, is 97320.

Required difference $=$ Largest number - Smallest number
= 97320-20379
$=76941$
Thus, the difference between the largest and the smallest numbers formed by the digits $0,9,3,7,2$, using each digit only once, is 76941.

Example : What is the correct descending order of the numbers 82931, 82382, 89231, 8293 and 81392?

## Sol.

Writing the given numbers in place value table.

| Ten thousands <br> $(\mathbf{T} \mathbf{~ T h})$ | Thousands <br> $(\mathbf{T})$ | Hundreds <br> $\mathbf{( H )}$ | Tens <br> $(\mathbf{T})$ | Ones <br> $(\mathbf{O})$ |
| :---: | :---: | :---: | :---: | :---: |
| 8 | 2 | 9 | 3 | 1 |
| 8 | 2 | 3 | 8 | 2 |
| 8 | 9 | 2 | 3 | 1 |
|  | 8 | 2 | 9 | 3 |
| 8 | 1 | 3 | 9 | 2 |

Arranging numbers in descending order means arranging them in decreasing order.
It can be observed that 8293 is the smallest among the given numbers.
The numbers $82931,82382,89231$ and 81392 have the same digit i.e. 8 at the Ten thousands place.
The digits at the Thousands place of these numbers are $2,2,9$ and 1 respectively. As $1<2<9$, therefore, 89231 is the greatest and 81392 is the smallest among these numbers.
Also, 82931 and 82382 have 9 and 3 respectively at the Hundreds place.
As $9>3$, therefore 82931 is greater than 82382 .
Thus, the correct descending order of the given numbers is, $89231,82931,82382,81392$ and 8293.

Example : Which of the following numbers is the greatest?
$97,342,97,432,9,734$ or $94,723$.

## Sol.

Writing the given numbers in the place value table as :

| Ten thousands <br> $(\mathbf{T}$ Th) | Thousands <br> $(\mathbf{T h})$ | Hundreds <br> $\mathbf{( H )}$ | Tens <br> $(\mathbf{T})$ | Ones <br> $\mathbf{( \mathbf { O } )}$ |
| :---: | :---: | :---: | :---: | :---: |
| 9 | 7 | 3 | 4 | 2 |
| 9 | 7 | 4 | 3 | 2 |
|  | 9 | 7 | 3 | 4 |
| 9 | 4 | 7 | 2 | 3 |

It can be observed from the table that 9,734 is the smallest number among all.
Now, comparing digits at various places. It can be observed that 97, 342, 97, 432 and 94, 723 have the same digit at the Ten thousands place i.e. 9.
Also, 97, 342 and 97, 432 have the same digit i.e. 7 at the Thousands place while 94, 723 have 4 at the Thousands place. As $7>4$. Therefore, 97,342 and 97,432 are greater than 94, 723.
97, 342 and 97,432 have 3 and 4 respectively at the Hundreds place. As $4>3$, therefore, 97,432 is greater than $97,342$.
Thus, 97,432 is the greatest among the given numbers.

## Multiple Choice Questions

1. What is the predecessor of the greatest six-digit number?
A. 100000
B. 100001
C. 999998
D. 999999
2. What is difference between the place values of two 5's in the number 4598351?
A. 499950
B. 49995
C. 49950
D. 0
3. $9,87,61,230$ $\square$ 9,87,16,230

Which of the following signs can be placed in the box between the two numbers?
A. $>$
B. $<$
C. $=$
D. None of these
4. 10000001 is the successor of $\qquad$
A. 10000002
B. 10000000
C. 9999999
D. None of these
5. How many three-digit numbers can be formed with the digits 3,0 and 7 without repetition?
A. 6
B. 5
C. 4
D. 3
6. Which are the respective greatest and the smallest numbers amongst 321987, 319240, 321978 and 321970?
A. 319240 and 321987
B. 321987 and 319240
C. 321987 and 321970
D. 319240 and 321978
7. What is the difference between the greatest and smallest 5 -digit numbers formed by using all the digits $3,0,9,1$ and 5 ?
A. 93951
B. 84951
C. 81720
D. 79172
8. Which of the following numbers is equal to 3 crore?
A. 3 million
B. 30 million
C. 300 million
D. 3000 million
9. How can the number fifty million twenty-one thousand two hundred thirty six be written using commas according to Indian system of numeration?
A. $50,021,236$
B. $5,00,21,236$
C. $50,00,21,236$
D. $500,021,236$
10. How can the numbers 10 million, 1 billion and 216 thousand be arranged in descending order?
A. 216 thousand, 10 million, 1 billion
B. 10 million, 216 thousand, 1 billion
C. 1 billion, 10 million, 216 thousand
D. 216 thousand, 1 billion, 10 million
11. $2,357,822$ $\square$ 2,357,799
A. $<$
B. $>$
C. $=$
D. None of these
12. The number 35 million ends with how many zeroes?
A. Four
B. Five
C. Six
D. Seven
13. Which one of the following numbers is prime?
A. 18
B. 19
C. 20
D. 21
14. Which one of the following numbers is prime?
A. 26
B. 27
C. 28
D. 29
15. Which one of the following numbers is composite?
A. 67
B. 69
C. 71
D. 73
16. Which one of the following numbers is composite?
A. 101
B. 103
C. 105
D. 107
17. The number 24 is to be written as a product of its prime factors. Which one of the following is correct?
A. $24=3 \times 8$
B. $24=4 \times 6$
C. $24=2 \times 3 \times 4$
D. $24=2 \times 2 \times 2 \times 3$
18. The number 90 is to be written as a product of its prime factors. Which one of the following is correct?
A. $90=2 \times 5 \times 9$
B. $90=2 \times 3 \times 3 \times 5$
C. $90=3 \times 5 \times 6$
D. $90=2 \times 3 \times 15$
19. The place value of 5 in 780756 is
A. Five ones
B. Five tens
C. 5 tenths
D. Five hundreds
20. Mach the following numbers in list I with the corresponding place value of number 5 .

Unit-I
(a) $750 \quad 1.500$
(b) 17510
(c) 124605
(d) 50630
abcd
A. 1234
B. 2134
C. 1234
D. 3124

## Answer Key

| 1. C | 2. A | 3. A | 4. B | 5. C | 6. B | 7. B | 8. B |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 9. B | 10. C | 11. B | 12. C | 13. B | 14. D | 15. B | 16. C |
| 17. D | 18. B | 19. B | 20. B |  |  |  |  |

1. The greatest six-digit number is 999999 . Therefore, predecessor of the greatest six-digit number is

$$
999999-1=999998
$$

Hence, the correct answer is C.
2. The given number is $45,98,351$.
$\therefore$ Place value of 5 at Lakhs place in the given number $=5 \times 100000$

$$
=500000
$$

Place value of 5 at Tens place in the given number $=5 \times 10=50$
$\therefore$ Difference between the place values of two 5 's in the given number $=500000-50=499950$
Hence, the correct answer is A.
3. The given numbers are 9, 87, 61, 230 and $9,87,16,230$. Both of these numbers have 8 digits.
These two numbers can be written in the place value table as follows:

| Crores | Ten <br> Lakhs | Lakhs | Ten <br> Thousands | Thousands | Hundreds | Tens | Ones |
| :---: | :---: | :---: | :--- | :---: | :---: | :---: | :---: |
| 9 | 8 | 7 | 6 | 1 | 2 | 3 | 0 |
| 9 | 8 | 7 | 1 | 6 | 2 | 3 | 0 |

Comparing the digits at ten thousands place, $6>1$.
9, 87, 61, $230>9,87,16,230$.
Therefore, the sign " $>$ " can be placed in the box.
Hence, the correct answer is A.
4. Successor of a number is the number that comes next to the given number and it can be obtained by adding 1 to the given number.
So, 10000001 is the successor of 10000000.

Hence, the correct answer is B.
5. The three-digit numbers that can be formed with the digits 3,0 and 7 can be listed as 307, 370, 703, 730.
Thus, 4 three-digit numbers can be formed with the given digits without repetition.
Hence, the correct answer is C.
6. The given numbers can be placed in the place value table as follows:

| Lakhs | Ten thousands | Thousands | Hundreds | Tens | Ones |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 2 | 1 | 9 | 8 | 7 |
| 3 | 1 | 9 | 2 | 4 | 0 |
| 3 | 2 | 1 | 9 | 7 | 8 |
| 3 | 2 | 1 | 9 | 7 | 0 |

It can be observed that the digits at lakhs place are the same.
On comparing the digits at ten thousands place, we get $2>1$.
$\therefore 319240$ is smallest amongst the given numbers.
Consider the numbers 321987,321978 and 321970.
Here, the digits at thousands place are the same.
Also, the digits at hundreds place are the same.
On comparing the digits at tens place, we get $8>7$.
$\therefore 321987>(321978,321970)$
Therefore, among the given numbers, 321987 and 319240 are the greatest and smallest numbers respectively Hence, the correct answer is B .
7. The given digits are $3,0,9,1$ and 5 . The greatest 5 -digit number formed by using all the given digits is 95310 .
The smallest 5-digit number formed by using all the given digits is 10359 .
$\therefore$ Required difference
$=95310-10359=84951$
Hence, the correct answer is B .
8. It is known that 1 million $=10$ lakh and

1 crore $=100$ lakh
$\therefore 1$ crore $=10 \times 10$ lakh $=10$ million
So, 3 crore $=3 \times 10$ million
$=30$ million
Therefore, 30 millions is equal to 3 crore.
Hence, the correct answer is B.
9. Fifty million twenty-one thousand two hundred and thirty-six can be written in International system of numeration as 50,021,236.
This number can be rewritten in Indian system of numeration as 5,00,21,236.
Hence, the correct answer is B .
10. 1 billion $=1000$ million
and 1 million $=1000$ thousands It is known that

$$
1000>10,10000>216
$$

$\therefore 1000$ million $>10$ million, 10000 million > 216 million
So, 1 billion $>10$ million, 10 million $>216$ thousand
$\therefore 1$ billion $>10$ million $>216$ thousand
Thus, the given numbers can be arranged in descending order as 1 billion, 10 million, 216 thousands Hence, the correct answer is C.
11. The numbers $2,357,822$ and $2,357,799$ can be placed in place value table as

| Million | Hundred <br> thousands | Ten <br> thousands | Thousands | Hundreds | Tens | Ones |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 5 | 7 | 8 | 2 | 2 |
| 2 | 3 | 5 | 7 | 7 | 9 | 9 |

It is seen that the digits at million, hundred thousands, ten thousands and thousands place are the same.
Comparing the digits at hundreds place, $8>7$.
Thus, $2,357,822>2,357,799$
Hence, the correct answer is B.
12. It is known that 1 million $=1,000,000$ 35 million $=35,000,000$
Therefore, there are six zeroes in 35 million.
Hence, the correct answer is C.
13.18 is composite because 18 can be written as a product of factors, not including 1 and 18 :

$$
18=2 \times 9 \text { or } 18=3 \times 6
$$

19 is prime because 19 doesn't have any factors except 1 and 19.
20 is composite because 20 can be written as a product of factors, not including 1 and 20 :

$$
20=2 \times 10 \text { or } 20=4 \times 5
$$

21 is composite because 21 can be written as a product of factors, not including 1 and 21:21 $=3 \times 7$
14.26 is composite because 26 can be written as a product of factors, not including 1 and 26 :

$$
26=2 \times 13
$$

27 is composite because 27 can be written as a product of factors, not including 1 and 27:

$$
27=3 \times 3 \times 3 \text { or } 3^{3}
$$

28 is composite because 28 can be written as a product of factors, not including 1 and 28:

$$
28=4 \times 7 \text { or } 2^{2} \times 7
$$

29 is prime because 29 doesn't have any factors except 1 and 29.
15. 69 is composite number. 67,71 and 73 are all prime numbers.
16. $105=3 \times 35$ or $3 \times 5 \times 7$, and so is composite.
101,103 and 107 are all prime numbers.
17.2 and 3 are prime numbers; 4,6 and 8 are composite numbers.
All the products are correct, but only answer D is a product of prime factors
18. 2, 3 and 5 are prime numbers; 6,9 and 15 are composite numbers.

All the products are correct, but only answer $B$ has a product of prime factors.
19. Place value of 5 is $5 \times 10=50$
20. (b) Place value of 5 in $750=5 \times 10=50$
$17510=5 \times 100=500$
$124605=5 \times 1=5$ $50630=5 \times 10000=50000$

## Unit-2 : Roman Numbers



Learning Objectives : In this unit, we will learn about:

- Rules for writing Roman Numerals
- How to convert to Roman Numerals?
- Rules on Roman Numbers


## Roman Numerals

Roman Numerals are originated in the ancient Roman Empire and are still in use today in a variety of forms. Roman numerals were adapted from Etruscan numerals. The system of Roman numerals that was used in classical antiquity was somewhat modified in the Middle Ages in order to produce the system that is used today. This system is based on certain letters which are given numerical value.
Use of roman numerals can still be seen in our daily life. On the face of some clocks, the hours are indicated by Roman Numerals. These numerals are used to do note different volumes of a book, questions in a question paper of exercise. Class room in a school.


## Did you know?

What the numbers 0123456789 are called?
These are Arabic numbers as these numbers came from Arabia. But apart from these numbers, we use some symbols too, which represent mathematical values. These numbers are the roman numerals.

