## IMO

## INTERNATIONAL MATHEMATICS OLYMPIAD



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We wish you all success in the examination and a very bright future in the field of mathematics.
All the best

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Section 1 Mathematical REASONING

## Unit-1: Number System



Learning Objectives : In this unit, we will learn about:

- Numbers (Numerals)
- Number Names
- 5 Digit Numbers
- Place Value
- Comparison of Numbers
- Successor and Predecessor
- Ascending and Descending Order
- Even and Odd Number


## Numbers (Numerals)

We are surrounded by numbers in each and every sphere of our life. Large numbers are often used in monetary transactions in businesses, banks, etc. Total numbers of schools in a city, total numbers of students in a university are all examples of large numbers.

## Number Names

Let us have a look at the table given below :

| Number | Number Name |
| :--- | :--- |
| 1 | One |
| 10 | Ten |
| 100 | One hundred |
| 1000 | One thousand |
| 10000 | Ten thousand |
| 100000 | One lakh |
| 1000000 | Ten lakh |
| 10000000 | Crore |
| 100000000 | Ten crore |

Numbers given above in the table are based on Indian System of Numeration. As the number increases, it becomes larger and larger.
As we know there are ten digits: $0,1,2,3,4,5,6,7,8$ and 9 . Numbers are written using these digits. These digits are called ones. The numerals formed by the digits $0,1,2,3,4$, ... are known as Hindu-Arabic numbers. This system is popular world-wide.
Numeral system is a way of counting and naming number. Number is an idea whereas the symbols used to represent the numbers are called numerals.

## 5 Digit Numbers

We know that 99,999 is the greatest 5 digit number. If we add 1 to it, it will give us the smallest 6 digit number.

## Place Value

Place value of a digit depends on its position in the number. As the digit moves to the left, its value increases.
The place value of a 6 digit number is Lakhs in the place value chart. Here is the relative chart :

| Lakhs Period |  | Thousands Period |  | Ones Period |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\stackrel{\sim}{0}$ |  |  |  |  |
| $\stackrel{\sim}{5}$ |  | $\begin{aligned} & \mathscr{O} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{F} \end{aligned}$ | n | O |  |  |
| $\stackrel{\smile}{\square}$ | 先 | $\stackrel{\smile}{\Perp}$ | $\begin{aligned} & \text { n } \\ & \text { ó } \\ & \end{aligned}$ | $\begin{aligned} & \overline{0} \\ & \overline{3} \\ & \text { 1 } \end{aligned}$ | $\stackrel{\sim}{\square}$ | $\stackrel{\text { ® }}{0}$ |
| 0 | 4 | 5 | 7 | 2 | 8 | 3 |

The place value chart has been separated into three groups: The ones period has three places - Hundreds, tens and ones. The thousands period has two places - Ten thousands and thousands.
Next period is the lakhs period which includes - Ten lakhs and lakhs.

## Comparison of Numbers

| Symbol | Meaning | Example in Symbols | Example in Words |
| :---: | :--- | :---: | :--- |
| $>$ | Greater than <br> More than <br> Bigger than <br> Larger than | $7>4$ | 7 is greater than 4 <br> 7 is more than 4 <br> 7 is bigger than 4 <br> 7 is larger than 4 |


| $<$ | Less than <br> Fewer than <br> Smaller than | $4<7$ | 4 is less than 7 <br> 4 has fewer than 7 <br> 4 is smaller than 7 |
| :--- | :--- | :---: | :--- |
| $=$ | Equal to <br> Same as | $7=7$ | 7 is equal to 7 <br> 7 is the same as 7 |

## Use of Commas

If we write the number without using the place values chart, we use comma to separate the periods.
Let us take an example : 4, 57, 283
First comma is used when the ones period is complete. Second comma is used when thousands period is complete. Next comma is used to separate thousands and lakhs period.

## Comparison of two numbers

If a number has more digits than the other, then it is greater of the two.
For example 8552 is greater than 285.
(i) If two numbers have the same number of digits and the extreme left digits are also the same then compare the next digits to the right and so on.
For example 342 is grater than 332.
(ii) If two numbers have the same number of digits then the number with bigger digit on the extreme left is greater. For example, 5732 is greater than 3584.
Note : Count the digits first, then check ' H ' then ' T ' then ' O '.

## Successor and Predecessor

## Successor

The number that comes just after a given number is called its Successor.

## Example :

Number
578
284
999

Successor
579
285
1000

The successor of a number is obtained by adding 1 to that number.

## Predecessor

The number that cames just before a particular number is called its predecessor.
Example :

Number
178
195
285

Predecessor
177
194
284

Clearly the predecessor of a number is obtained by subtracting 1 from the given number. Note : Zero has no predecessor.

## Ascending and Descending Order

## Ascending order

Arranging the given numbers from the smallest to the greatest is called ascending order or increasing order.
Example : Arrange these numbers in ascending order
4572, 5132, 4698, 8455
Sol. In 4572, 5132, 4698, 8455
As $4572<4698<5132<8455$
$\therefore$ Ascending order is $4572,4698,5132,8455$

## Descending order

Arranging the given number from the greatest to the smallest is called descending order or decreasing order.
Example : Arrange these numbers in descending order.
5431, 3451, 5231, 4531
Sol. In 5431, 3451, 5231, 4531
As $5431>523174531>3451$
$\therefore$ Decreasing order is 5431, 5231, 4531, 3451 .

## Even and Odd Number

## Even numbers

In an even number the digits in the ones place is $0,2,4,6$ or 8 .

## Odd numbers

In an odd number the digits in the ones place is $1,3,5,7$ or 9 .

## Expanded form of 9999

$$
\begin{aligned}
9999 & =9 \text { thousands }+9 \text { hundreds }+9 \text { tens }+9 \text { ones } \\
& =9 \times 1000+9 \times 100+9 \times 10+9 \times 1 \\
& =9000+900+90+9
\end{aligned}
$$

## Multiple Choice Questions

1. 99,999 is the greatest number.
A. 5
B. 4
C. 3
D. 2
2. Place value of 5 in $5,43,621$ is $\qquad$ .
A. Lakh
B. Thousand
C. Ones
D. Tens
3. Smallest 6 digit number $=$ $\qquad$ .
A. 0,00,000
B. 1,00,000
C. $9,99,999$
D. 99,999
4. $300000+20000+4000+200+2=$
$\qquad$ .
A. $3,24,202$
B. $3,42,222$
C. $2,34,222$
D. $3,22,432$
5. Ones period includes:
A. Hundreds
B. Thousands
C. Ten thousands
D. Lakhs
6. Lakhs period includes:
A. Thousand
B. Lakh
C. Tens
D. Ones
7. Thousands period includes:
A. Ten thousands
B. Tens
C. Ones
D. Hundreds
8. Pick the odd one out :
A. Hundreds
B. Tens
C. Ones
D. Urans
9. We use $\qquad$ to separate the periods.
A. Comma
B. Full stop
C. Brackets
D. Hyphen
10. $4,37,283$ is a $\qquad$ number.
A. 4 digit
B. 5 digit
digit
C. 6 digit
D. 7 digit
11. Instead of putting comma, we can to separate the periods.
A. Put hyphen
B. Leave space
C. Put full stop
D. Put brackets
12. Ten lakhs comes in $\qquad$ period.
A. Thousands
B. Lakhs
C. Ones
D. Hundreds
13. Pick odd one out:
A. $6,34,231$
B. $1,34,345$
C. $1,34,655$
D. $1,23,456$
14. Pick the odd one out:
A. $6,44,245$
B. $4,65,345$
C. 2,55,666
D. $2,566,55$
15. Pick the odd one out:
A. 1,00,000
B. 10,000
C. 1,00,001
D. $10,00,00$
16. $3,44,567$ has $\qquad$ lakhs.
A. 3
B. 4
C. 5
D. 6
17. $3,44,567$ has $\qquad$ thousands
A. 3
B. 4
C. 5
D. 6
18. $3,44,567$ has $\qquad$ ones
A. 3
B. 4
C. 7
D. 5
19. $3,44,567$ has $\qquad$ tens
A. 3
B. 4
C. 5
D. 6
20. $3,44,567$ has $\qquad$ hundreds
A. 3
B. 4
C. 5
D. 6

## Answer Key

1. A
2. A
3. B
4. A
5. A
6. B
7. A
8. D
9. A
10. C
11. B
12. B
13. D
14. D
15. D
16. A
17. B
18. C
19. D
20. C

## Hints and Solutions

1. Greatest 5 digit number $=99999$
2. Place value of 5 in $5,43,621$

$$
=5 \times 100,000=500000
$$

3. Smallest 6 digit number $=1,00,000$
4. We have $300000+20000+4000+$ $200+2$

$=$| L | TTh | T | H | T | O |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 0 | 2 | 0 | 0 | 0 | 0 |
| 0 | 0 | 4 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 2 |
|  | 3 | 2 | 4 | 2 | 0 |

9. We use comma to separate the periods.
10. Instead of putting comma, we can leave space to separate the periods.
11. $1,23,456$ is an even number.
12. 3,44,567 has 3 lakhs.

Since L TTh T H T O
$\begin{array}{llllll}3 & 4 & 4 & 5 & 6 & 7\end{array}$

## Unit-2 : Roman Numerals



Learning Objectives : In this unit, we will learn about:

- Roman Symbols
- Uses of Roman Symbols

Roman numbers are used widely in our daily life. The most important and common example is watches and clocks with Roman numbers on it. Roman numerals are used to number different volumes of a book classroom in a school. Questions in a question paper or exercise.


## Roman Symbols

There are seven symbols used in this system which are as follows :

$$
I, V, X, L, C, D \& M
$$

## Value of the symbol

Each symbol has a corresponding value:

| Roman Symbols |  |  |
| :--- | :--- | :--- |
| I | stands for | 1 |
| V | stands for | 5 |
| X | stands for | 10 |
| L | stands for | 50 |
| C | stands for | 100 |
| D | stands for | 500 |
| M | stands for | 1000 |

Note : There is no symbol for zero in the roman numeral system.

## Uses of Roman Numerals

1. When certain roman numerals are repeated, the value of the resulting numeral is equal to their sum.

$$
\begin{aligned}
& I I I=1+1+1=3 \\
& X X=10+10=20
\end{aligned}
$$

2. Roman numerals read from left to right, larger values to the left and work to the smaller values on the right.
3. If a lesser symbol is before a greater symbol, the lesser is subtracted from the greater. For example, IV = 5-1=4
4. If a lesser symbol is after a greater symbol, the two values are added. For example, $\mathrm{VI}=5+1=6$
5. I and $V$ can only modify up to an $X$. For example, 49 is not written IL, rather you first resolve 40 as XL and then resolve 9 as IX. Put them together and $49=10+$ $40+9$ = XLIX.
6. $X$ and $L$ can only modify up to a C. For example, 490 are not written XD. First you resolve 400 as CD and then you resolve 90 as XC. Put them together and $490=$ CDXC.
7. C and D can only modify up to an M. For example, 950 is not written LM, rather you first resolve 900 as CM and then add L for 50 . So $950=\mathrm{CML}$.
Note : (a) V, L, D are not repeated (b) No roman numeral can come together more than three times. It is wrong to write IIII $=4$. (c) The symbol V can never be written on the left of any greater value symbol.

## Shortcuts to Problem Solving

1. An accurate way to write the roman numbers is to first take the thousands, hundreds, tens and ones.
Example : 1999, One thousand is M, nine hundred is CM, ninety is XC, nine is IX. Combine all these : MCMXCIX
2. Develop a mnemonic device to remember the order of Roman numerals. Think "MeDiCaLXaVIer". It has the roman numerals in order from 1000 to 1.


Another common mnemonic like "I Value Xylo-phones Like Cows Dig Milk" puts the Roman numerals I, V, X, L, C, D and M in order from smallest to largest. If you have problem only in remembering larger numbers, it may help you to remember that " C " is equivalent to "century" and " M " is equivalent to "millennium": 100 and 1000 , respectively.
3. Write the six pairs of subtractive Roman numeral on a notecard along with their equivalents in Roman numerals, "IV" is equal to 4 , "IX" is equal to 9 , "XL" is equal to 40 , " XC " is equal to 90 , " CD " is equal to 400 and " CM " is equal to 900 . These are called "subtractive" because the first letter is "subtracted" from the second. Keep the notecard visible at all times so you know to recognize these pairs when they appear.
Example : Shraddha wants to convert her friend's year of birth(1989) into roman numbers. Can you help her to do so?
Sol. Break 1989 into 1000, 900, 80 and 9, then do each conversion.
$1000=M \quad 900=C M$
$80=$ LXXX $\quad 9=I X$
So, $1989=1000+900+80+9=$ MCMLXXXIX

